

A Novel Semantic Similarity Measure based on overlap between concepts for Semantic Web Services Matching and Composition

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Abstract

Similarity/dissimilarity measurement plays a crucial role in information/component/service retrieval and integration. In this paper, we define a novel logic based semantic similarity/dissimilarity measure that tries to estimate a pseudo ideal similarity metric based on Description Logic based descriptions of concepts in ontologies in order to fulfill the requirements for similarity measurement in the field of web service retrieval (i.e. web services matching and composition). Our proposed semantic similarity measure defined and represented as a set of rules, considers the direction in comparing two concepts which may be from different ontologies and it computes the similarity/dissimilarity between two concepts by the extent to which the second concept includes instances which are also included by the first concept. It can handle the expressivity of highly expressive Description Logic based ontology languages such as OWL DL to a considerable extent. We also conceptually compare our proposed measure with other similar methods in the target application area.

Keywords: Inter-concept Similarity/Dissimilarity, Overlap between Concepts, Description Logics, Ontology, Similarity Measure, Semantic Matching, Web Service Composition

1. Introduction

In the presented research, we have focused on a sort of logic based semantic similarity/dissimilarity measures that is suitable to be used for semantic matching of web services in order to automatically compose new web services by discovering and integrating the existing ones. For automating the process of web services matching and composition, we need to semantically describe web services using ontology languages. Currently, web services besides their syntactic specifications or descriptions by standard syntactic models such as WSDL, are semantically specified or described by standard semantic models such as OWL-S and SAWSDL [16, 35]. Intuitively, an effective matching of web services involves considering all of their functional and non-functional requirements specified in their descriptions and interfaces, but the most crucial part of web

services matching is their signatures matching (i.e. inputs, outputs, preconditions, and effects). So, in our research we have only focused on web services input/output matching for web service composition when the inputs and outputs of web services have been semantically described in ontologies [2, 13, 16, 18, 23, 25, 29].

So far, many similarity measures have been proposed in the literature [25]. In our research, we have only focused on logic based semantic similarity measures. As demonstrated in our previous research paper [25], in order to improve the process of semantic matching of web services, the semantic similarity/dissimilarity measures generally need to measure the extent to which the second concept includes instances which are also included by the first concept. Logic based semantic similarity measures can be perfect for computing the similarity between concepts in the field of web service retrieval if and only if they can adequately handle the expressivity of the used ontology language in order to estimate the pseudo ideal similarity metric introduced in Section 2 [25]. Hence, in this paper, we propose a logic based semantic similarity measure as a set of descriptor-specific rules which tries to estimate the ideal similarity metric as precise as possible based on OWL descriptions of concepts in ontologies.

In the next section, we review the theoretical background to our research regarding Web Services, Ontologies, and Description Logics (DL). In Section 3, we define and present our novel DL-based semantic similarity/dissimilarity measure. In Section 4, we conceptually discuss and compare the proposed measure with other methods. Finally in Section 5, we conclude this paper with several issues that need further investigation and the future works.

2. Background: Web Services, Ontologies, and Description Logics

In our previous work presented in [25], we investigated the problem of matching web services with each other in order to integrate them and compose a new web service. We analyzed such a complicated problem in order to deduce the requirements for similarity measurement in the field of web service retrieval. Our analysis led to defining a “pseudo ideal similarity metric” which ideally expresses the requirements for similarity measurement in the field of web service retrieval using mathematical symbols whereas it cannot be generally actually computed. Because it is defined using operators of set theory such as union, intersection, difference, and specifically the cardinality or size of sets that are not directly

computable since they are applied to ontology concepts that are considered as subsets of the interpretation domain objects i.e. Δ^I that is undetermined. The ideal similarity metric was defined as follows [25]:

$$DD(A, B, I) = \frac{|A^I \cup B^I|}{|A^I \cap B^I|} * \frac{|A^I - B^I|}{|A^I|}; \quad (2.1)$$

$$MHD(A, B, I) = \text{Minimum hierarchical distance between } A \text{ and } B; \quad (2.2)$$

$$\text{Dissim}(A, B, I) = DD(A, B, I) + MHD(A, B, I); \quad (2.3)$$

$$\text{Sim}(A, B, I) = \frac{\mu}{\mu + \text{Dissim}(A, B, I)}; \quad \mu > 0 : \text{adjustable factor} \quad (2.4)$$

Where A and B are concepts from the same or different ontologies. Based on an interpretation like $I = (\Delta^I, \varphi^I)$ for the ontology (or ontologies), A is mapped to A^I and B is mapped to B^I . $|A|$ denotes the size of the set A . $\text{Sim}(A, B, I)$ and $\text{Dissim}(A, B, I)$ denote the semantic similarity and dissimilarity from A to B respectively. $MHD(A, B, I)$ denotes the minimum hierarchical distance between A and B in the ontological hierarchy after classification. $\text{Dissim}(A, B, I)$ and $\text{Sim}(A, B, I)$ can be converted to each other using Equation 2.4. μ is an adjustable factor [25].

We defined the pseudo ideal similarity metric to only make our approach to similarity measurement more understandable by specifying the semantics of similarity (values) based on which the similarity has to be computed by web service matchmakers/composers in the field of web service retrieval. In other words, in order to improve the process of semantic matching of web services, the used semantic similarity/dissimilarity measure has to try to estimate the introduced ideal similarity metric that means it has to compute the similarity between concepts by the extent to which the second concept includes instances which are also included by the first concept. When we speak about the estimation of such a pseudo ideal measure, we just mean doing computations which lead to the values with the semantics represented by this pseudo ideal measure considering what it really tries to measure. So, comparing a computable logic based similarity measure with this ideal measure in order to investigate how much imprecision we get, is meaningless. In other words, we can only compare computable logic based measures with each other to investigate how well they are able to estimate the ideal measure relative to each other. It is clear that such similarity/dissimilarity measures are asymmetric and consider the direction. Also, such a matching approach considers the state in which two concepts (classes)

overlap, as a level of match higher than the disjoint level even if none of the two concepts subsumes the other. Most of approaches to semantic matching are based on subsumption reasoning and do not consider such states as a level of match [2, 5, 10, 16, 21, 23, 38]. Also, a good similarity measure has to be able to handle the expressivity of the description logics used for describing web service parameters i.e. web service Inputs, Outputs, Preconditions, and Effects (IOPEs) in ontologies [16, 17, 28]. Hence, in the presented research, we have focused on the problem of computing the similarity/dissimilarity between two concepts in order to define a novel sophisticated computable logic based similarity/dissimilarity measure which tries to estimate the pseudo ideal similarity metric based on DL based descriptions of concepts in ontologies and therefore it is suitable to be used in semantic matching of web services for web service composition.

In some research papers, the existing semantic similarity/dissimilarity measures have been divided into measures for concepts from the same ontology and measures for concepts from different ontologies [37, 40]. A fully-automatic method for computing the similarity/dissimilarity between concepts from different ontologies is desired, but our proposed measure, presented in the next section, is based on a semi-automatic approach for concepts from different ontologies since the experts of the ontologies need to check and investigate the subsumption relations between properties and primitive concepts from different ontologies and then set some probability parameters for configuring the proposed measure (Please refer to Section 3 for details). So, our proposed measure has been devised in a way that is able to compute the similarity/dissimilarity between concepts from the same or different ontologies both.

The semantic similarity/dissimilarity between concepts is computed based on their semantic descriptions in ontologies. Thus far many ontology languages have been proposed and standardized such as RDF(S) and OWL for defining concepts and their conceptual relations in ontologies. Despite the apparent differences, many of the current ontology languages can be regarded as tractable and decidable subsets of Description Logics. In our research, we narrowed our focus to OWL ontology language, because *Description logics* form its formal foundation and OWL has been being endorsed by the semantic web initiative [22]. In 2009, W3 Consortium produced a recommendation for a new version of OWL which adds features to the 2004 version, while remaining compatible. Some of the new

features are syntactic sugar while others offer new expressivity, including: keys, property chains, richer datatypes and data ranges, qualified cardinality restrictions, and asymmetric, reflexive, and disjoint properties [22]. While handling the expressivity of more expressive DL-based ontology languages such as OWL 2 is desired for computing the similarity/dissimilarity between concepts, but it can be achieved by long term, ongoing research efforts [11, 12, 28].

In this paper, we present a similarity/dissimilarity measure which tries to handle the expressivity of OWL DL to a considerable extent. OWL DL is one of the three sub-species of OWL 1 and it is based on *SHOIN* DL. Let CN denotes a concept name, C and D are arbitrary concepts, R is a property, n is a non-negative integer, d and o_i ($1 \leq i \leq n$) are instances, and T and \emptyset denote the top (i.e. Thing) and the bottom (i.e. empty class) respectively. Then, a *SHOIN* concept is [9, 22, 28]:

$$CN \mid C \sqcap D \mid C \sqcup D \mid \neg C \mid \exists R.C \mid \forall R.C \mid \exists R.d \mid =_n R.T \mid \geq_n R.T \mid \leq_n R.T \mid \{o_1, \dots, o_n\}$$

OWL Construct	Formal Representation
owl:equivalentClass	$C_1 \equiv C_2$
owl:disjointWith	$C_1 \perp C_2$
owl:complementOf	$C_1 \equiv \neg C_2$
owl:subClassOf	$C_1 \sqsubseteq C_2$
owl:intersectionOf	$C_1 \sqcap C_2$
owl:unionOf	$C_1 \sqcup C_2$
owl:minCardinality	$\geq_n R.T$
owl:maxCardinality	$\leq_n R.T$
owl:cardinality	$=_n R.T$
owl:allValuesFrom	$\forall R.D$
owl:someValuesFrom	$\exists R.D$
owl:hasValue	$\exists R.d$

Table 2.1 – OWL constructs and their formal representations

In Table 2.1, the formal representations of the most important constructs in OWL ontology language are shown [9, 28]. In this table, C_1 , C_2 , and D are concepts (classes), R is a property, T is the top (Thing), d is an instance, and n is a non-negative integer.

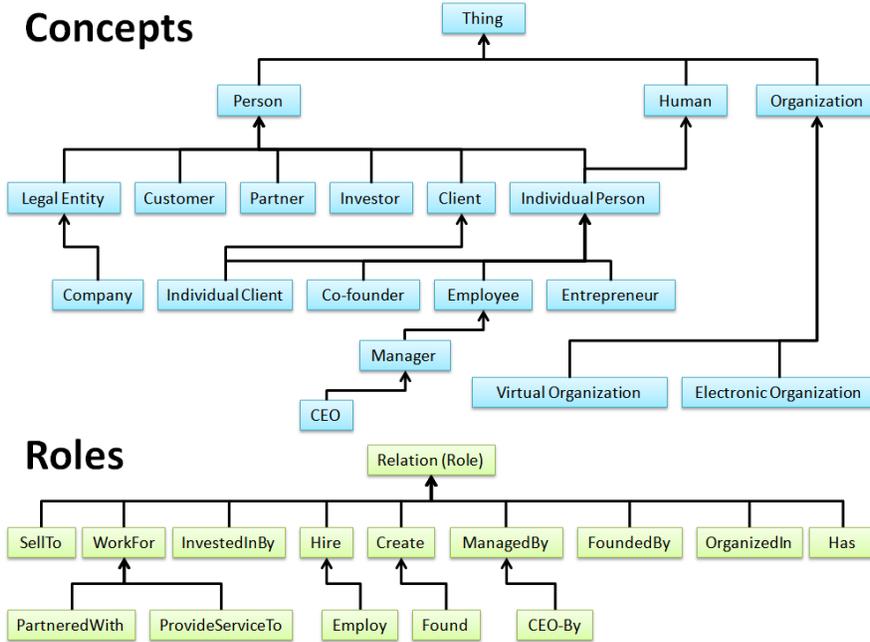
3. The proposed similarity/dissimilarity measure

3.1. Case Study

In order to make the rules more understandable, we present a case study as a detailed example on how the introduced rules are applied to compute the

similarity/dissimilarity between concepts. The hierarchical structure of the exemplary ontology has been depicted in Picture 3.1.

Assume that: $A \equiv \text{Organization}$, $B \equiv \text{Legal Entity}$, $D \equiv \text{Client}$, $E \equiv \text{Company}$, $R \equiv \text{WorkFor}$, $S \equiv \text{ProvideServiceTo}$, $d_1 \equiv \text{TPS IT Company}$, $d_2 \equiv \text{Turan Designers Inc.}$, $d_3 \equiv \text{Iran Cell Inc.}$ So, we have (T is the ontology root i.e. Thing): $\text{MHD}(D, T) = 2$, $\text{MHD}(E, T) = 3$, $\text{MHD}(E, D) = 1$, $S \sqsubset R$, $\text{MHD}(R, S) = 1$. Also, We do not have any explicit or inferred statement in the ontology representing any of the following facts: $E \sqsubseteq D$, $D \sqsubseteq E$, $A \sqsubseteq B$, $D \sqsubseteq B$. Also, for simplification we consider the ranges of S and R to be T.



Picture 3.1 – The hierarchical structure of an ontology which is used for demonstrating the applicability of our proposed semantic similarity measure.

Assume that CN_1 is an Organization which operates as a Legal Entity or as a client in Iran. It only works for companies. It works for at least one entity and works for at least one company. It provides some services to TPS IT Company, and also provides some services to Turan Designers Inc. We can logically define CN_1 as follows:

$$CN_1 = A \sqcap (B \sqcup D) \sqcap \forall R. E \sqcap \exists R. E \sqcap \exists S. d_1 \sqcap \exists S. d_2 \sqcap (\geq_1 R. T)$$

Assume that CN_2 is an Organization which operates as a Legal Entity or a non-organized group. It only provides services to companies and only works for clients. It provides services to at least five entities. It also provides services

to at least one client. It also works for Iran Cell Inc. We can logically define CN_2 as follows:

$$CN_2 = A \sqcap (B \sqcup \neg A) \sqcap \forall S.E \sqcap \forall R.D \sqcap \exists S.D \sqcap \exists R.d_3 \sqcap (\geq_5 S.T)$$

Also, we assume: $MHD(A, CN_1) = 3$, $MHD(A, CN_2) = 2$, $MHD(B, CN_1) = 4$, $MHD(B, CN_2) = 2$.

3.2. Definition of an alignment for concept descriptors

After the canonization process introduced and proposed in our previous research paper [25], each generated model of a concept description can be represented in a canonical form as follows (Please refer to [25] for details and explanations):

$$\begin{aligned} \mathcal{L}'(x_{CN_i}) = & \{ C_{i1}, C_{i2}, \dots, C_{in_i}, [\forall R_1.G_{i1}]_1, [\forall R_2.G_{i2}]_1, \dots, [\forall R_k.G_{ik}]_1, \\ & [\exists R_1.H_{i1} \mid \exists R_1.d_{i1}]_\infty, [\exists R_2.H_{i2} \mid \exists R_2.d_{i2}]_\infty, \dots, [\exists R_k.H_{ik} \mid \exists R_k.d_{ik}]_\infty, \\ & [\geq_{m_1} R_1.T \mid \mid_{=s_1} R_1.T]_1, [\geq_{m_2} R_2.T \mid \mid_{=s_2} R_2.T]_1, \dots, [\geq_{m_k} R_k.T \mid \mid_{=s_k} R_k.T]_1, \\ & [\leq_{r_1} R_1.T \mid \mid_{=s_1} R_1.T]_1, [\leq_{r_2} R_2.T \mid \mid_{=s_2} R_2.T]_1, \dots, [\leq_{r_k} R_k.T \mid \mid_{=s_k} R_k.T]_1, \\ & [-D]_1, \{ \{ o_1, \dots, o_p \} \}_1 \} \quad (3.1) \end{aligned}$$

And the two compared concepts CN_1 and CN_2 can be represented as follows:

$$CN_1 = (\mathcal{L}'(x_{CN_{11}}), \mathcal{L}'(x_{CN_{12}}), \dots, \mathcal{L}'(x_{CN_{1n}}))$$

$$CN_2 = (\mathcal{L}'(x_{CN_{21}}), \mathcal{L}'(x_{CN_{22}}), \dots, \mathcal{L}'(x_{CN_{2m}}))$$

Where $\mathcal{L}'(x_{CN_{1i}})$ ($1 \leq i \leq n$) and $\mathcal{L}'(x_{CN_{2j}})$ ($1 \leq j \leq m$) are the models generated for the two compared concepts CN_1 and CN_2 respectively. In our approach, each model of the first concept will be compared with every model of the second concept, and then among the $m \times n$ results obtained for similarity/dissimilarity, the result with the least value for dissimilarity or the most value for similarity will be used for computing the final similarity/dissimilarity value between the two concepts.

Each model of a concept CN can be also regarded as a specific interpretation or a specific subset of the interpretations of the original description of CN . So, when we select two models from the model sets of the two compared concepts in order to compare them with each other, in fact we select two specific interpretations or two specific subsets of the interpretations of the two concepts to compare them with each other. Hence, when we finally choose the result with the least value for dissimilarity from all the results

obtained from comparing the models of the two concepts, in fact we choose those interpretations which lead to the least value for dissimilarity of the two concepts. We stated this fact to illustrate the relation between our proposed computable measure and the ideal measure introduced in Section 2 since the definition of the ideal measure shows that similarity/dissimilarity depends on the interpretation chosen for ontologies [25].

Assume that $Dissim(CN_1, CN_2)$ is the dissimilarity from CN_1 to CN_2 , $Dissim(\mathcal{L}'(x_{CN_1i}), \mathcal{L}'(x_{CN_2j}))$ is the dissimilarity from $\mathcal{L}'(x_{CN_1i})$ to $\mathcal{L}'(x_{CN_2j})$ that can be shortly represented as $Dissim_{ij}$ which is computed using the descriptor-specific rules of our proposed measure presented in Section 3.3, and $MHD(\mathcal{L}'(x_{CN_1i}), \mathcal{L}'(x_{CN_2j}))$ is the minimum hierarchical distance between $\mathcal{L}'(x_{CN_1i})$ and $\mathcal{L}'(x_{CN_2j})$ in the ontological hierarchy after classification that it can be shortly represented as MHD_{ij} . Then we define $Dissim(CN_1, CN_2)$ as follows:

$$Dissim(CN_1, CN_2) = \text{Min}_{1 \leq i \leq n, 1 \leq j \leq m} (Dissim_{ij} + MHD_{ij}) \quad (3.2)$$

If CN_1 and CN_2 are from the two different ontologies O_1 and O_2 respectively and $T(O_1)$ and $T(O_2)$ are the roots of these two ontologies, then MHD_{ij} can be computed as follows:

$$MHD_{ij} = |MHD(\mathcal{L}'(x_{CN_1i}), T(O_1)) - MHD(\mathcal{L}'(x_{CN_2j}), T(O_2))| \quad (3.3)$$

Or a more perfect computational scheme be used based on the depth or granularity of the respective ontological hierarchies. In our research, we have not focused on such alignment schemes and leave it with the above simple scheme. If CN_1 and CN_2 are from the same ontologies, then MHD_{ij} is computed by the following equation (S is a primitive concept existing in both compared models):

$$MHD_{ij} = \text{Min}_{S \in \mathcal{L}'(x_{CN_1i}), S \in \mathcal{L}'(x_{CN_2j})} (|MHD(S, CN_1) - MHD(S, CN_2)|) \quad (3.4)$$

We use MHD_{ij} in Equation (3.2) to differentiate two concepts which cannot be adequately differentiated just by comparing their semantic descriptions in ontologies [28]. For instance, if we have $A \sqsubset B \sqsubset C$, then the semantic dissimilarity between A and C must be more than the dissimilarity between A and B . But these may not be adequately differentiated by the values

computed for Dissim_{ij} . For example, if A, B, and C are primitive concepts, we are not able to differentiate them using the descriptor-specific rules of our measure presented in Section 3.3. Hence, MHD complements our logic based structural measure particularly in situations that concepts are not adequately described in ontologies. We may also need to convert the dissimilarity to the similarity using an equation like the one below [37] which was also introduced in the definition of our pseudo ideal measure in Section 2 [25]:

$$\text{Sim}(CN_1, CN_2) = \frac{\mu}{\mu + \text{Dissim}(CN_1, CN_2)}, \quad \mu > 0 : \mu \text{ is an adjustable factor. (3.5)}$$

In this step, we also need to specify how the various types of descriptors from two compared models are aligned for comparison. In our approach, descriptors which have more effect on the logical interpretation of each other are considered for comparison with each other. In this way, primitive concepts and negations (i.e. disjoint concepts) are considered as the first category of descriptors to be compared with each other, the value and existential restrictions represented as $\forall R.G$, $\exists R.G$, and $\exists R.d$ are considered as the second category, and cardinality related restrictions represented as $\geq_n R.T$, $\leq_n R.T$, or $=_n R.T$ are considered as the third category. Our proposed measure is not able to handle the descriptors of properties (roles) such as ones which are described as transitive, symmetric, functional, or inverse functional properties while they may affect the logical interpretation of other types of descriptors. Such descriptors are a part of some expressive DL-based ontology languages such as OWL-DL, but handling them based on our introduced ideal measure is beyond the scope of this research. We do not also claim that descriptors from different categories do not have any effect on the logical interpretation of each other, but we naturally want and need to reasonably make simple the complicated problem of computing similarity/dissimilarity between concepts. Such a categorization can be justified considering the facts that each type of descriptors states. For instance, the value and existential restrictions state facts about the values of properties while the cardinality related restrictions say facts about the number of properties with the same type that any instance of a concept can have. The alignment explained above, has been reflected in the descriptor specific rules presented in the next section. To explicitly say, the condition parts of those rules somewhat reflect the

aforementioned alignment and show which types of descriptors are considered for comparison with each other.

Considering our case study, first we need to convert the concept descriptions into their canonical form and find all the possible models of them using the canonization rules presented in our previous research paper [25]:

$$\mathcal{L}'(x_{CN_1,1}) = \{A, B, \forall R.E, \exists R.E, \ni S.d_1, \ni S.d_2, \geq_1 R.T\}$$

$$\mathcal{L}'(x_{CN_1,2}) = \{A, D, \forall R.E, \exists R.E, \ni S.d_1, \ni S.d_2, \geq_1 R.T\}$$

$$\mathcal{L}'(x_{CN_1,3}) = \{A, B, D, \forall R.E, \exists R.E, \ni S.d_1, \ni S.d_2, \geq_1 R.T\}$$

$$\mathcal{L}'(x_{CN_2,1}) = \{A, B, \forall R.D, \forall S.(D \sqcap E), \exists S.(D \sqcap E), \ni R.d_3, \geq_5 S.T\}$$

$$\mathcal{L}'(x_{CN_2,2}) = \{A, -A, \forall R.D, \forall S.(D \sqcap E), \exists S.(D \sqcap E), \ni R.d_3, \geq_5 S.T\} \#(\text{clash}:A, -A)$$

$$\mathcal{L}'(x_{CN_2,3}) = \{A, B, -A, \forall R.D, \forall S.(D \sqcap E), \exists S.(D \sqcap E), \ni R.d_3, \geq_5 S.T\} \#(\text{clash}:A, -A)$$

The $\mathcal{L}'(x_{CN_2,2})$ and $\mathcal{L}'(x_{CN_2,3})$ models are not satisfiable, so they are ignored within the next step where we use the descriptor-specific rules of our proposed measure to compare the models of the two concepts.

3.3. Application of descriptor specific dissimilarity functions

We have followed heuristic methods to invent a number of rules which can be collectively used for estimating the pseudo ideal measure introduced in Section 2 [25]. We justify these rules by explaining how they try to estimate the introduced ideal measure from their particular descriptor-specific perspectives. Considering these rules, overall dissimilarity between the two models $\mathcal{L}'(x_{CN_1,i})$ and $\mathcal{L}'(x_{CN_2,j})$ (i.e. Dissim_{ij}) is gradually computed by executing the rules presented in this section that handle various types of descriptors. α_i ($1 \leq i \leq 14$), M_1 , M_2 , β_1 , and β_2 are adjustable factors which have been used in the equations of the rules and explained in detail at the end of this section. Most of these rules are asymmetric relative to the two compared models, because we want to measure the extent to which the second model includes instances which are also included by the first model and not vice versa. In the equations of these rules, the value of $\frac{a}{b}$ with $a > 0$ and $b = 0$, is considered as infinite (∞), the value of $\frac{a}{\beta^b}$ with $a = \infty$, $b = \infty$, and

$\beta > 1$, is considered as 0, and the value of $a * b$ with $a = \infty$ and $b = 0$, is considered as 0.

First, let us explain the notion behind all of these rules. Essentially, each descriptor in the semantic description of a concept makes some restrictions for the domain instances in being instances of that concept so that some instances can be instances of that concept and some others cannot be instances of that concept. By using the rules presented in this step, we compare descriptors from the descriptions of two concepts in order to determine whether a descriptor in the description of the second concept in comparison with a descriptor in the description of the first concept, makes the instances set defined by the second concept more restricted than the instances set defined by the first concept or not, and if so, how much it is more restricted. So, by comparing two concept descriptions in this manner, we actually try to estimate the introduced ideal measure based on logic based descriptions of concepts in ontologies. As examples, consider the following states:

- 1) If there is a min-cardinality restriction like $\geq_m R.T$ in the description of the first concept and a min-cardinality restriction like $\geq_n R.T$ in the description of the second concept and we have $m < n$, then we can deduce that the second concept is more restricted than the first concept from the perspective of these two descriptors and as a result, such descriptors reduce the chance for the second concept to include more instances which are also included by the first concept. This is also true if there is a max-cardinality restriction like $\leq_m R.T$ in the description of the first concept and a max-cardinality restriction like $\leq_n R.T$ in the description of the second concept and we have $m > n$. Also, if there is a min-cardinality restriction like $\geq_m R.T$ in the description of one concept and a max-cardinality restriction like $\leq_n R.T$ in the description of the other concept and we have $m > n$, then the two concepts cannot have any common instance and are disjoint. Rule 10 handles such states in comparing two role cardinality related descriptors.
- 2) If there is a value restriction like $\forall R.A$ in the description of the first concept and a value restriction like $\forall R.B$ in the description of the second concept and we have $B \sqsubset A$, then we can deduce that the second concept

is more restricted than the first concept from the perspective of these two descriptors and as a result, such descriptors reduce the chance for the second concept to include more instances which are also included by the first concept. Rules 6, 7 and 8 handle such states in comparing two value or existential restriction descriptors.

- 3) The states will be more complicated if we compare role restrictions with two different roles like S and R when $S \sqsubset R$, for examples when we compare $\geq_m R.T$ and $\geq_n S.T$, or when we compare $\forall R.A$ and $\forall S.B$. For instance, if we have $\geq_n S.T$ and $S \sqsubseteq R$, then we can logically deduce that $\geq_n R.T$, but if $R \sqsubset S$, we cannot deduce any minimum cardinality restriction on R considering $\geq_n S.T$. Hence, when we compare $\geq_m R.T$ from the description of the first concept and $\geq_n S.T$ from the description of the second concept, in the first state (i.e., $S \sqsubseteq R$), the two restrictions are logically interconnected with respect to our introduced ideal measure since we can deduce $\geq_n R.T$ from $\geq_n S.T$, and therefore considering the fact that $\geq_n R.T$ may restrict the second concept more than what $\geq_m R.T$ restricts the first concept (i.e., if $n > m$), we have to compare the two descriptors, but in the second state (i.e., $R \sqsubset S$), since we cannot deduce any fact about minimum cardinality of R from $\geq_n S.T$, comparison between the two descriptors cannot show whether the second concept is more restricted than the first concept from the perspective of these two descriptors or not, hence in the second state (i.e., $R \sqsubset S$), the comparison does not have to be done. This fact can also be considered in the case of value restrictions. Because when we have $\forall R.A$ and $S \sqsubseteq R$, then we can logically deduce that $\forall S.A$, but if $R \sqsubset S$, we cannot deduce any value restriction on S from $\forall R.A$. Rule 5 has been devised for comparing roles with respect to subsumption relation between them. The dissimilarity values between roles, resulted from executing Rule 5, are used in Rules 6, 7, 8, 9, and 10 which handle various types of restrictions on roles and are also able to handle the complexities exemplified above.

Each descriptor-specific rule tries to estimate the introduced ideal measure from the perspective of some specific types of descriptors. Rule 1 compares the primitives of two models with respect to subsumption relation.

Rule 2 compares the primitives of two models with respect to disjoint relation. Rule 3 compares the primitives of one model with disjoints of the other one. Rule 4 compares the disjoints of two models. Rule 5 compares roles with respect to subsumption relation. Rule 6 compares the value restrictions (i.e., \forall) of two models. Rule 7 compares the value restrictions of one model with existential restrictions (i.e., \exists) of the other one. Rule 8 compares the existential restrictions of two models. Rule 9 compares the property-value existence restrictions (i.e., \ni) of two models. Rule 10 compares the role cardinality related restrictions (i.e., \geq_{m_k} & \leq_{r_k} & $=_{s_k}$) of two models. Rule 11 compares the enumerations of two models. In order to use the rules, we need to assign values to the adjustable factors. So, we assign the following values to them in our case study:

$$\alpha_1 = 3, \alpha_2 = 1, \alpha_3 = 7, \alpha_4 = 5, \alpha_5 = 1, \alpha_6 = 2, \alpha_7 = \alpha_8 = 1, \alpha_9 = \alpha_{10} = 2, \\ \alpha_{11} = \alpha_{12} = \alpha_{13} = \alpha_{14} = 1, \beta_1 = 0.5, \beta_2 = 2, M_1 = 5, M_2 = 15$$

The above assignment is based on our intuition and the mathematical structure of the equations used in the rules as discussed at the end of this section.

Rule 0 - Action: $\text{Dissim}_{ij} := 0$

Considering our example: $\text{Dissim}_{11} = 0$; $\text{Dissim}_{21} = 0$; $\text{Dissim}_{31} = 0$;

Rule 1 (Subsumption between primitives)

$$\left. \begin{array}{l} C_1 \in \mathcal{L}'(x_{CN_2j}) \\ \delta_{C_1} = P(\nexists C_2 . (C_2 \sqsubseteq C_1) \in \mathcal{L}'(x_{CN_1i})) \\ C_1, C_2: \text{primitive} \\ P(s) \text{ is the probability function.} \end{array} \right\} \Rightarrow \text{Dissim}_{ij} := \text{Dissim}_{ij} + \alpha_1 \delta_{C_1}, \quad \alpha_1 \geq 0$$

Explanation and Justification:

C_1 is a primitive concept in the definition of the $\mathcal{L}'(x_{CN_2j})$ model. δ_{C_1} is the probability for non-existence of any primitive concept like C_2 in the definition of the $\mathcal{L}'(x_{CN_1i})$ model that is subsumed by C_1 and such a subsumption either has been explicitly stated in the ontology or can be inferred. If the two compared concepts CN_1 and CN_2 are from the same ontology, we have $\delta_{C_1} = 1$ or $\delta_{C_1} = 0$, because we can make sure of the existence or non-existence of a concept like $C_2 \in \mathcal{L}'(x_{CN_1i})$ that $C_2 \sqsubseteq C_1$ by

reasoning or considering the explicit statements of the ontology, otherwise δ_{C_1} can be determined by experts of the two ontologies. Non-existence of any concept like $C_2 \in \mathcal{L}'(x_{CN_1i})$ that $C_2 \sqsubseteq C_1$, can be interpreted as the less chance for $\mathcal{L}'(x_{CN_2j})$ to include more instances which are also included by $\mathcal{L}'(x_{CN_1i})$, and therefore we add the value of $\delta_{C_1} \alpha_1$ to Dissim_{ij} to increase the dissimilarity between models. This rule is executed once for each primitive concept in the definition of the second model. Considering our exemplary ontology, we have ($P(s)$ is the probability function):

$$\begin{aligned} \text{Dissim}(E, T) &= \text{Dissim}(\mathcal{L}'(x_E), \mathcal{L}'(x_T)) + \text{MHD}(\mathcal{L}'(x_E), \mathcal{L}'(x_T)) = \\ &= \alpha_1 P(E \not\sqsubseteq T) + \text{MHD}(E, T) = \alpha_1 \times 0 + 3 = 3; \end{aligned}$$

$$\begin{aligned} \text{Dissim}(T, D) &= \text{Dissim}(\mathcal{L}'(x_T), \mathcal{L}'(x_D)) + \text{MHD}(\mathcal{L}'(x_T), \mathcal{L}'(x_D)) = \\ &= \alpha_1 P(T \not\sqsubseteq D) + \text{MHD}(D, T) = \alpha_1 \times 1 + 2 = 5; \end{aligned}$$

$$\begin{aligned} \text{Dissim}(T, (D \sqcap E)) &= \text{Dissim}(\mathcal{L}'(x_T), \mathcal{L}'(x_{D \sqcap E})) + \text{MHD}(\mathcal{L}'(x_T), \mathcal{L}'(x_{D \sqcap E})) \\ &= \alpha_1 P(T \not\sqsubseteq D) + \alpha_1 P(T \not\sqsubseteq E) + \text{MHD}(T, (D \sqcap E)) = \alpha_1 \times 1 + \alpha_1 \times 1 + 3 = 9; \end{aligned}$$

$$\begin{aligned} \text{Dissim}(E, D) &= \text{Dissim}(\mathcal{L}'(x_E), \mathcal{L}'(x_D)) + \text{MHD}(\mathcal{L}'(x_E), \mathcal{L}'(x_D)) = \\ &= \alpha_1 P(E \not\sqsubseteq D) + \text{MHD}(E, D) = 3 \times 1 + 1 = 4; \end{aligned}$$

$$\begin{aligned} \text{Dissim}(E, (D \sqcap E)) &= \text{Dissim}(\mathcal{L}'(x_E), \mathcal{L}'(x_{D \sqcap E})) + \text{MHD}(\mathcal{L}'(x_E), \mathcal{L}'(x_{D \sqcap E})) \\ &= \alpha_1 P(E \not\sqsubseteq D) + \alpha_1 P(E \not\sqsubseteq E) + \text{MHD}(E, (D \sqcap E)) = \alpha_1 \times 1 + \alpha_1 \times 0 + 1 = 4; \end{aligned}$$

$$\begin{aligned} \text{Dissim}((D \sqcap E), E) &= \text{Dissim}(\mathcal{L}'(x_{D \sqcap E}), \mathcal{L}'(x_E)) + \text{MHD}(\mathcal{L}'(x_{D \sqcap E}), \mathcal{L}'(x_E)) \\ &= \alpha_1 P(E \not\sqsubseteq E \wedge D \not\sqsubseteq E) + 1 = \alpha_1 \times 0 + 1 = 1; \end{aligned}$$

$$\text{Dissim}_{21} := \text{Dissim}_{21} + \alpha_1 P(A \not\sqsubseteq B \wedge D \not\sqsubseteq B) = \alpha_1 \times 1 = 3;$$

Rule 2 (disjoint primitives)

$$\left. \begin{array}{l} C_1 \in \mathcal{L}'(x_{CN_1i}) \\ \delta'_{C_1} = P(\exists C_2. (C_1 \sqsubseteq -C_2) \in \mathcal{L}'(x_{CN_2j})) \\ C_1, C_2 : \text{primitive} \\ P(s) \text{ is the probability function.} \end{array} \right\} \Rightarrow \text{Dissim}_{ij} := \text{Dissim}_{ij} + \frac{\alpha_2 \delta'_{C_1}}{1 - \delta'_{C_1}}, \alpha_2 \geq 0$$

Explanation and Justification:

C_1 is a primitive concept in the definition of the $\mathcal{L}'(x_{CN_1i})$ model. δ'_{C_1} is the probability for existence of a primitive concept like C_2 in the definition of the

$\mathcal{L}'(x_{CN_2j})$ model that is disjoint with C_1 . If the two compared concepts CN_1 and CN_2 are from the same ontology, we have $\delta'_{C_1} = 1$ or $\delta'_{C_1} = 0$, because we can make sure of the existence or non-existence of a concept like $C_2 \in \mathcal{L}'(x_{CN_2j})$ that $C_1 \sqsubseteq -C_2$ by reasoning or considering the explicit statements of the ontology, otherwise δ'_{C_1} can be determined by experts of the two ontologies. If there is a concept like $C_2 \in \mathcal{L}'(x_{CN_2j})$ that $C_1 \sqsubseteq -C_2$, then the two models $\mathcal{L}'(x_{CN_1i})$ and $\mathcal{L}'(x_{CN_2j})$ do not have any common instance and they are disjoint, so according to our ideal measure, the dissimilarity between them must be infinite. The same result is accordingly yielded using this rule. If $\delta'_{C_1} = 1$, then Dissim_{ij} is infinite, and if $\delta'_{C_1} = 0$, then nothing is added to the previous value of Dissim_{ij} . Generally, we add the value of $\frac{\alpha_2 \delta'_{C_1}}{1 - \delta'_{C_1}}$ to the previous value of Dissim_{ij} . This rule is executed once for each primitive concept in the definition of the first model.

Rule 3 (primitives & disjoints)

$$\left. \begin{array}{l} \left(C \in \mathcal{L}'(x_{CN_1i}) \wedge \exists -D \in \mathcal{L}'(x_{CN_2j}) \right) \\ \vee \left(C \in \mathcal{L}'(x_{CN_2j}) \wedge \exists -D \in \mathcal{L}'(x_{CN_1i}) \right) \\ C: \text{primitive} \end{array} \right\} \Rightarrow \text{Dissim}_{ij} := \text{Dissim}_{ij} + \frac{\alpha_3}{\text{Dissim}(C, D) - \text{MHD}(C, D)}, \alpha_3 \geq 0$$

Explanation and Justification:

Considering the two models $\mathcal{L}'(x_{CN_1i})$ and $\mathcal{L}'(x_{CN_2j})$ and a primitive concept like C in the definition of one of the two models, if there is a disjoint like $-D$ in the definition of the other model, then the value of $\frac{\alpha_3}{\text{Dissim}(C, D) - \text{MHD}(C, D)}$ is added to the previous value of Dissim_{ij} . The less value of $\text{Dissim}(C, D) - \text{MHD}(C, D)$ can be interpreted as the less chance for $\mathcal{L}'(x_{CN_2j})$ to include more instances which are also included by $\mathcal{L}'(x_{CN_1i})$, and therefore the more the value which has to be added to the previous value of Dissim_{ij} . For instance, if C is subsumed by D , then the two models $\mathcal{L}'(x_{CN_1i})$ and $\mathcal{L}'(x_{CN_2j})$ do not have any common instance and they are disjoint, so according to our introduced ideal measure, the dissimilarity between them must be infinite. The same result is

accordingly yielded using this rule. $\text{Dissim}(C, D)$ is computed using our descriptor specific rules. Considering Equation 3.2, we always have $\text{Dissim}(C, D) \geq \text{MHD}(C, D)$, and considering the canonical form of the description of models shown in Equation 3.1, there is at most one concept like D that $-D \in \mathcal{L}'(x_{CN_2j})$ or $-D \in \mathcal{L}'(x_{CN_1i})$, so if there is a disjoint like $-D$ in the definition of one of the two models, for each primitive concept in the definition of the other model, this rule must be executed once.

Rule 4 (disjoints)

$$\left. \begin{array}{l} \exists -D \in \mathcal{L}'(x_{CN_1i}) \\ \exists -D' \in \mathcal{L}'(x_{CN_2j}) \end{array} \right\} \Rightarrow \text{Dissim}_{ij} := \text{Dissim}_{ij} + \alpha_4(1 - \beta_1^{\text{Dissim}(D, D')})$$

$$, \quad 0 \leq \beta_1 \leq 1, \quad \alpha_4 \geq 0$$

Explanation and Justification

If there is a concept like D that $-D \in \mathcal{L}'(x_{CN_2j})$ and also a concept like D' that $-D' \in \mathcal{L}'(x_{CN_2j})$, then the value of $\alpha_4(1 - \beta_1^{\text{Dissim}(D, D')})$ is added to the previous value of Dissim_{ij} . The more value of $\text{Dissim}(D, D')$ can be interpreted as the less chance for $\mathcal{L}'(x_{CN_2j})$ to include more instances which are also included by $\mathcal{L}'(x_{CN_1i})$, and therefore the more the value which has to be added to the previous value of Dissim_{ij} . Since β_1 is not more than 1, the value of $\alpha_4(1 - \beta_1^{\text{Dissim}(D, D')})$ is at most equal to α_4 . Such a treatment supports and is compatible with our introduced ideal measure because even if D and D' are disjoint, it is still possible that the two models have instances in common. $\text{Dissim}(D, D')$ is computed using our descriptor specific rules. Considering the canonical form of the description of models previously shown in Equation 3.1, there is at most one concept like D that $-D \in \mathcal{L}'(x_{CN_1i})$ and at most one concept like D' that $-D' \in \mathcal{L}'(x_{CN_2j})$. Hence, this rule can be executed once at most.

Rule 5 (roles)

Assume that $R_{k'_1}$ and $R_{k'_2}$ are two properties (roles) from two ontologies O_1 and O_2 respectively. If $R_{k'_1}$ and $R_{k'_2}$ are from the same ontology, then we have $O_1 = O_2$. There are properties like R_{k_1} and R_{k_2} in the ontological hierarchy of properties from the ontologies O_1 and O_2 respectively

that do not have any superproperty and they also subsume $R_{k'_1}$ and $R_{k'_2}$ respectively ($R_{k'_1} \sqsubseteq R_{k_1}$, $R_{k'_2} \sqsubseteq R_{k_2}$). $\delta_{k'_1 k'_2}$ is the probability for $R_{k'_1} \sqsubseteq R_{k'_2}$, and $\delta_{k'_2 k'_1}$ is the probability for $R_{k'_2} \sqsubseteq R_{k'_1}$. So, we have $\delta_{k'_1 k'_2} + \delta_{k'_2 k'_1} \leq 1$. Then the semantic dissimilarity between $R_{k'_1}$ and $R_{k'_2}$ represented as $\text{RDissim}(R_{k'_1}, R_{k'_2})$ is computed by the following rule:

$$\left. \begin{array}{l}
 R_{k_1}, R_{k'_1} \in O_1, R_{k'_1} \sqsubseteq R_{k_1} \\
 R_{k_2}, R_{k'_2} \in O_2, R_{k'_2} \sqsubseteq R_{k_2} \\
 \nexists R_{k'} \cdot (R_{k_1} \sqsubset R_{k'}) \in O_1, \nexists R_{k'} \cdot (R_{k_2} \sqsubset R_{k'}) \in O_2 \\
 \delta_{k'_1 k'_2} = P(R_{k'_1} \sqsubseteq R_{k'_2}), \delta_{k'_2 k'_1} = P(R_{k'_2} \sqsubseteq R_{k'_1})
 \end{array} \right\} \Rightarrow$$

$$\text{RDissim}(R_{k'_1}, R_{k'_2}, \rho, \theta) := \alpha_{k'_1 k'_2} \left(\frac{\theta \alpha_5 \delta_{k'_1 k'_2} + \rho \alpha_6 \delta_{k'_2 k'_1} + 1 - \delta_{k'_1 k'_2} - \delta_{k'_2 k'_1}}{\delta_{k'_1 k'_2} + \delta_{k'_2 k'_1}} \right)$$

$$, \alpha_{k'_1 k'_2} = 1 - \delta_{k'_1 k'_2} - \delta_{k'_2 k'_1} + |\text{MHD}(R_{k_1}, R_{k'_1}) - \text{MHD}(R_{k_2}, R_{k'_2})|$$

$$, 0 \leq \alpha_5 \leq \alpha_6, \rho = \text{empty}(\rho) ? 1 : \rho, \theta = \text{empty}(\theta) ? 1 : \theta$$

$$, O_1 = O_2 \Rightarrow |\text{MHD}(R_{k_1}, R_{k'_1}) - \text{MHD}(R_{k_2}, R_{k'_2})| \approx \text{MHD}(R_{k'_1}, R_{k'_2})$$

$\text{MHD}(R_k, R_{k'})$ is the minimum hierarchical distance between R_k and $R_{k'}$ in the ontological hierarchy. The default value for ρ and θ is 1 when they are not provided as arguments. α_5 and α_6 are adjustable factors.

Explanation and Justification:

As it has been shown in Rules 7 and 10, the ρ and θ factors can be used as multiplier or negator of the generally determined α_6 and α_5 factors respectively if needed. As it is clear, the two factors α_5 and α_6 can make $\text{RDissim}(R_{k'_1}, R_{k'_2})$ asymmetric relative to $R_{k'_1}$ and $R_{k'_2}$. Only if $\alpha_5 = \alpha_6$, and $\theta = \rho$, then $\text{RDissim}(R_{k'_1}, R_{k'_2})$ is symmetric. The less the probability for $R_{k'_1} \sqsubseteq R_{k'_2}$ or the less the probability for $R_{k'_2} \sqsubseteq R_{k'_1}$, the more the dissimilarity between $R_{k'_1}$ and $R_{k'_2}$ is and if these probabilities tend to 0, then $\text{RDissim}(R_{k'_1}, R_{k'_2})$ tends to infinite (∞). The dissimilarity between two properties have to be infinite if none of them subsumes the other. The effect and role of this rule is illustrated where we present the rules which handle the restrictions on properties (roles). So, this rule supports and is compatible with our introduced ideal measure. If $R_{k'_1}$ and $R_{k'_2}$ are from the same ontology, then there are only three possible states: ($\delta_{k'_1 k'_2} = 0$ and $\delta_{k'_2 k'_1} = 0$) or ($\delta_{k'_1 k'_2} = 1$ and $\delta_{k'_2 k'_1} = 0$) or ($\delta_{k'_1 k'_2} = 0$ and $\delta_{k'_2 k'_1} = 1$), because we can

make sure of the correctness or incorrectness of $R_{k'_1} \sqsubseteq R_{k'_2}$ or $R_{k'_2} \sqsubset R_{k'_1}$ by reasoning or considering the explicit statements in the ontology, otherwise $\delta_{k'_1 k'_2}$ and $\delta_{k'_2 k'_1}$ can be determined by experts of the two ontologies.

$$\delta_{k'_1 k'_2} = 1 \Rightarrow \delta_{k'_2 k'_1} = 0 \Rightarrow \text{RDissim}(R_{k'_1}, R_{k'_2}) = \theta\alpha_5 |\text{MHD}(R_{k_1}, R_{k'_1}) - \text{MHD}(R_{k_2}, R_{k'_2})|$$

$$\xrightarrow{O_1=O_2} \text{RDissim}(R_{k'_1}, R_{k'_2}) = \theta\alpha_5 \text{MHD}(R_{k'_2}, R_{k'_1})$$

$$\delta_{k'_2 k'_1} = 1 \Rightarrow \delta_{k'_1 k'_2} = 0 \Rightarrow \text{RDissim}(R_{k'_1}, R_{k'_2}) = \rho\alpha_6 |\text{MHD}(R_{k_1}, R_{k'_1}) - \text{MHD}(R_{k_2}, R_{k'_2})|$$

$$\xrightarrow{O_1=O_2} \text{RDissim}(R_{k'_1}, R_{k'_2}) = \rho\alpha_6 \text{MHD}(R_{k'_2}, R_{k'_1})$$

$$R_{k'_1} = R_{k'_2} \Rightarrow \text{RDissim}(R_{k'_1}, R_{k'_2}) = 0.$$

Considering our example, we have:

$$\text{RDissim}(R, R) = 0;$$

$$\text{RDissim}(R, S) = \alpha_6 \text{MHD}(R, S) = 2 \times 1 = 2;$$

$$\text{RDissim}(S, R) = \alpha_5 \text{MHD}(R, S) = 1 \times 1 = 1;$$

Rule 6 (value restrictions \forall)

$$\left. \begin{array}{l} \forall R_k. G_{ik} \in \mathcal{L}'(x_{CN_1i}) \\ \exists (\forall R_{k'}. G_{jk'} \in \mathcal{L}'(x_{CN_2j})) \end{array} \right\} \Rightarrow \text{Dissim}_{ij} := \text{Dissim}_{ij} + \alpha_7 \frac{\text{Dissim}(G_{ik}, G_{jk'})}{\beta_2 \text{RDissim}(R_k, R_{k'})}, \quad 0 \leq \alpha_7, \quad 1 < \beta_2$$

Explanation and Justification:

Considering the canonical form of the description of models previously shown in Equation 3.1, for each property like R_k , there is at most one restriction like $\forall R_k. G_{ik}$ in the description of the $\mathcal{L}'(x_{CN_1i})$ model and for each property like $R_{k'}$, there is at most one restriction like $\forall R_{k'}. G_{jk'}$ in the description of the $\mathcal{L}'(x_{CN_2j})$ model. If the $\mathcal{L}'(x_{CN_1i})$ model does not have any \forall -type statement for a property like R , we add one to the description of that model as follows: $\forall R. A$ (A represents the range of the property R). This is done to make possible effective comparison between the two models from the perspective of all \forall -type statements in the definition of the $\mathcal{L}'(x_{CN_2j})$ model that make it more restricted than the $\mathcal{L}'(x_{CN_1i})$ model. According to the above rule, if there is a restriction like $\forall R_{k'}. G_{jk'}$ in the description of the $\mathcal{L}'(x_{CN_2j})$ model, for each restriction like $\forall R_k. G_{ik}$ in the description of the $\mathcal{L}'(x_{CN_1i})$ model, we have to add the value of

$\alpha_7 \frac{\text{Dissim}(G_{ik}, G_{jk'})}{\beta_2^{\text{RDissim}(R_k, R_{k'})}}$ to the previous value of Dissim_{ij} . The more value of $\text{Dissim}(G_{ik}, G_{jk'})$ if we have $\text{RDissim}(R_k, R_{k'}) \neq \infty$, can be interpreted as the less chance for $\mathcal{L}'(x_{CN_2j})$ to include more instances which are also included by $\mathcal{L}'(x_{CN_1i})$, and therefore the more the value which has to be added to the previous value of Dissim_{ij} . For instance, If CN_1 and CN_2 are from the same ontology, G_{ik} and $G_{jk'}$ are disjoint (i.e. $\text{Dissim}(G_{ik}, G_{jk'}) = \infty$), and $\text{RDissim}(R_k, R_{k'}) \neq \infty$, then the two models $\mathcal{L}'(x_{CN_1i})$ and $\mathcal{L}'(x_{CN_2j})$ do not have any common instance and they are disjoint. The same result is accordingly yielded using this rule. $\text{Dissim}(G_{ik}, G_{jk'})$ is computed using our descriptor specific rules. Considering our example, we have:

$$\text{Dissim}_{11} := \text{Dissim}_{11} + \alpha_7 \left(\frac{\text{Dissim}(E, D)}{\beta_2^{\text{RDissim}(R, R)}} + \frac{\text{Dissim}(T, (D \cap E))}{\beta_2^{\text{RDissim}(S, S)}} + \frac{\text{Dissim}(E, (D \cap E))}{\beta_2^{\text{RDissim}(R, S)}} + \frac{\text{Dissim}(T, D)}{\beta_2^{\text{RDissim}(S, R)}} \right) = 0 + 1 \times \left(\frac{4}{2^0} + \frac{9}{2^0} + \frac{4}{2^2} + \frac{5}{2^1} \right) = 16.5;$$

$$\text{Dissim}_{21} := \text{Dissim}_{21} + \alpha_7 \left(\frac{\text{Dissim}(E, D)}{\beta_2^{\text{RDissim}(R, R)}} + \frac{\text{Dissim}(T, (D \cap E))}{\beta_2^{\text{RDissim}(S, S)}} + \frac{\text{Dissim}(E, (D \cap E))}{\beta_2^{\text{RDissim}(R, S)}} + \frac{\text{Dissim}(T, D)}{\beta_2^{\text{RDissim}(S, R)}} \right) = 3 + 1 \times \left(\frac{4}{2^0} + \frac{9}{2^0} + \frac{4}{2^2} + \frac{5}{2^1} \right) = 19.5;$$

Rule 7 (value and existential restrictions \forall & \exists)

$$\left. \begin{array}{l} \forall R_k. G_{ik} \in \mathcal{L}'(x_{CN_1i}) \\ \exists (\exists R_{k'}. H_{jk'}) \in \mathcal{L}'(x_{CN_2j}) \end{array} \right\} \Rightarrow \text{Dissim}_{ij} := \text{Dissim}_{ij} + \alpha_8 \text{Max}_{(\exists R_{k'}. H_{jk'}) \in \mathcal{L}'(x_{CN_2j})} \left(\frac{\text{Dissim}(G_{ik}, H_{jk'})}{\beta_2^{\text{RDissim}(R_{k'}, R_k, \infty)}} \right), \quad 0 \leq \alpha_8, \quad 1 < \beta_2$$

Explanation and Justification:

Considering the canonical form of the description of models previously shown in Equation 3.1, for each property like R_k , there is at most one restriction like $\forall R_k. G_{ik}$ in the description of the $\mathcal{L}'(x_{CN_1i})$ model and if there is a restriction like $\exists R_{k'}. H_{jk'}$ in the description of the $\mathcal{L}'(x_{CN_2j})$ model, then according to this rule, we have to add the maximum value among the values of $\frac{\alpha_8 \text{Dissim}(G_{ik}, H_{jk'})}{\beta_2^{\text{RDissim}(R_{k'}, R_k, \infty)}}$ for all restrictions like $\exists R_{k'}. H_{jk'} \in \mathcal{L}'(x_{CN_2j})$, to the previous value of Dissim_{ij} . If $\text{RDissim}(R_{k'}, R_k, \infty) \neq \infty$, the more value of

Dissim(G_{ik} , $H_{jk'}$) can be interpreted as the less chance for $\mathcal{L}'(x_{CN_2j})$ to include more instances which are also included by $\mathcal{L}'(x_{CN_1i})$, and therefore the more the value which has to be added to the previous value of Dissim_{ij}, but if the probability for $R_k \sqsubset R_{k'}$ is not 0, then RDissim($R_{k'}$, R_k , ∞) is infinite (∞), and the value of $\frac{\alpha_8 \text{Dissim}(G_{ik}, H_{jk'})}{\beta_2 \text{RDissim}(R_{k'}, R_k, \infty)}$ is considered as 0. Such a treatment supports and is compatible with our ideal measure, because if $R_k \sqsubset R_{k'}$, the two restrictions $\forall R_k. G_{ik}$ and $\exists R_{k'}. H_{jk'}$ are not logically interconnected with respect to the introduced ideal measure and therefore they do not have to be compared, for instance even if $H_{jk'}$ and G_{ik} are disjoint, it is still possible that the two models have instances in common from the perspective of these two restrictions, but in the case that $R_{k'} \sqsubseteq R_k$, if $H_{jk'}$ and G_{ik} are disjoint, then the two models do not have any common instance. Dissim(G_{ik} , $H_{jk'}$) is computed using our descriptor specific rules, and RDissim($R_{k'}$, R_k , ∞) is computed using Rule 5. For each restriction like $\forall R_k. G_{ik} \in \mathcal{L}'(x_{CN_1i})$, this rule is executed once. Considering our example, we have:

$$\text{Dissim}_{11} := \text{Dissim}_{11} + \alpha_8 \text{Max} \left(\frac{\text{Dissim}(E, (D \cap E))}{\beta_2 \text{RDissim}(S, R, \infty)} \right) = 16.5 + \alpha_8 \left(\frac{4}{21} \right) = 18.5;$$

$$\text{Dissim}_{21} := \text{Dissim}_{21} + \alpha_8 \text{Max} \left(\frac{\text{Dissim}(E, (D \cap E))}{\beta_2 \text{RDissim}(S, R, \infty)} \right) = 19.5 + \alpha_8 \left(\frac{4}{21} \right) = 21.5;$$

Rule 8 (existential restrictions \exists)

$$\left. \begin{array}{l} \exists R_k. H_{ik} \in \mathcal{L}'(x_{CN_1i}) \\ \exists (\exists R_{k'}. H_{jk'}) \in \mathcal{L}'(x_{CN_2j}) \\ \text{RDissim}(R_k, R_{k'}) \neq \infty \end{array} \right\} \Rightarrow \text{Dissim}_{ij} := \text{Dissim}_{ij} + \alpha_9 (1 - \beta_1^{F_1})$$

$$, F_1 = \begin{cases} \text{Min}_{(\exists R_k. H_{ik}) \in \mathcal{L}'(x_{CN_1i})} \left(\text{Max}_{(\exists R_{k'}. H_{jk'}) \in \mathcal{L}'(x_{CN_2j})} \left(\frac{\text{Dissim}(H_{ik}, H_{jk'})}{\beta_2 \text{RDissim}(R_k, R_{k'})} \right) \right) \\ M_1 \quad \text{if } \nexists (\exists R_k. H_{ik}) \in \mathcal{L}'(x_{CN_1i}) \text{ and } \exists (\exists R_{k'}. H_{jk'}) \in \mathcal{L}'(x_{CN_2j}) \end{cases}$$

, $0 \leq \alpha_9$, $0 \leq \beta_1 \leq 1$, $1 < \beta_2$, M_1 : an adjustable factor with a relatively big value

Explanation and Justification

If there are restrictions like $\exists R_k. H_{ik}$ in the description of the $\mathcal{L}'(x_{CN_1i})$ model and there are restrictions like $\exists R_{k'}. H_{jk'}$ in the description of the $\mathcal{L}'(x_{CN_2j})$ model, then according to this rule, we have to find the minimum

value as F_1 , for all restrictions like $\exists R_k, H_{ik} \in \mathcal{L}'(x_{CN_1i})$, among the maximum values of $\frac{\text{Dissim}(H_{ik}, H_{jk'})}{\beta_2^{\text{RDissim}(R_k, R_{k'})}}$ for all restrictions like $\exists R_{k'}, H_{jk'} \in \mathcal{L}'(x_{CN_2j})$ with $\text{RDissim}(R_k, R_{k'}) \neq \infty$, and then add the value of $\alpha_9(1 - \beta_1^{F_1})$ to the previous value of Dissim_{ij} . If there is not any restriction like $\exists R_k, H_{ik}$ in the description of the $\mathcal{L}'(x_{CN_1i})$ model, but there are some in the description of the $\mathcal{L}'(x_{CN_2j})$ model, then $F_1 = M_1$. M_1 is an adjustable factor which can be a relatively big number. Since $0 \leq \beta_1 \leq 1$, we have $0 \leq \alpha_9(1 - \beta_1^{F_1}) \leq \alpha_9$. The value of $\alpha_9(1 - \beta_1^{F_1})$ is at most equal to α_9 when F_1 is infinite (∞), so there is an upper limit for the value which may be added to Dissim_{ij} by the action of this rule. Such a treatment is necessary because the high values for $\frac{\text{Dissim}(H_{ik}, H_{jk'})}{\beta_2^{\text{RDissim}(R_k, R_{k'})}}$ can not be directly considered as a criterion for determining the extent to which the $\mathcal{L}'(x_{CN_2j})$ model includes instances which are also included by the $\mathcal{L}'(x_{CN_1i})$ model. For instance, even if the values of $\text{Dissim}(H_{ik}, H_{jk'})$ for all restrictions like $\exists R_k, H_{ik} \in \mathcal{L}'(x_{CN_1i})$ and $\exists R_{k'}, H_{jk'} \in \mathcal{L}'(x_{CN_2j})$ are infinite, then it is still possible that the two models have instances in common. $\text{Dissim}(H_{ik}, H_{jk'})$ is computed using our descriptor specific rules and $\text{RDissim}(R_k, R_{k'})$ is computed using Rule 5. This rule is executed once at most. Considering our example, we have:

$$F_1 = \frac{\text{Dissim}(E, (D \cap E))}{\beta_2^{\text{RDissim}(R, S)}} = \frac{\text{Dissim}(E, (D \cap E))}{2^2} = \frac{4}{4} = 1;$$

$$\text{Dissim}_{11} := \text{Dissim}_{11} + \alpha_9(1 - \beta_1^{F_1}) = 18.5 + 2(1 - (\frac{1}{2})^1) = 19.5;$$

$$\text{Dissim}_{21} := \text{Dissim}_{21} + \alpha_9(1 - \beta_1^{F_1}) = 21.5 + 2(1 - (\frac{1}{2})^1) = 22.5;$$

Rule 9 (property-value existence restriction \exists)

$$\left. \begin{array}{l} \exists R_k, d_1 \in \mathcal{L}'(x_{CN_1i}) \\ \exists R_{k'}, d_{1'} \in \mathcal{L}'(x_{CN_2j}) \end{array} \right\} \Rightarrow \text{Dissim}_{ij} := \text{Dissim}_{ij} + \alpha_{10}(1 - \beta_1^{F_2})$$

$$F_2 = \sum_k \sum_{k'} \frac{|\{d_{1'} | \exists R_{k'}, d_{1'} \in \mathcal{L}'(x_{CN_2j})\} - \{d_1 | \exists R_k, d_1 \in \mathcal{L}'(x_{CN_1i})\}|}{\beta_2^{\text{RDissim}(R_{k'}, R_k)}}$$

, $0 \leq \alpha_{10}$, $0 \leq \beta_1 \leq 1$, $1 < \beta_2$

Explanation and Justification:

If for any pair of properties like $(R_k, R_{k'})$ with $\text{RDissim}(R_{k'}, R_k) \neq \infty$, there is an instance like $d_{1'}$ for which the $\exists R_{k'}.d_{1'}$ restriction exists in the description of the second model, but there is not any restriction like $\exists R_k.d_{1'}$ in the description of the first model, then this can be interpreted as the less chance for $\mathcal{L}'(x_{CN_2j})$ to include more instances which are also included by $\mathcal{L}'(x_{CN_1i})$ from the perspective of these two restrictions, and therefore the more the value which has to be added to the previous value of Dissim_{ij} . Such an interpretation can be realized by subtracting the $\{d_1 \mid \exists R_k.d_1 \in \mathcal{L}'(x_{CN_1i})\}$ set from the $\{d_{1'} \mid \exists R_{k'}.d_{1'} \in \mathcal{L}'(x_{CN_2j})\}$ set in the equation given for F_2 . This rule is executed once. Considering our example, we have:

$$F_2 = \frac{|\{d_3\} - \{d_1, d_2\}|}{\beta_2^{\text{RDissim}(R, S)}} = \frac{1}{\beta_2^{\text{RDissim}(R, S)}} = \frac{1}{2^2} = 0.25;$$

$$\text{Dissim}_{11} = \text{Dissim}_{11} + \alpha_{10}(1 - \beta_1^{F_2}) = 19.5 + 2(1 - (\frac{1}{2})^{0.25}) = 19.82;$$

$$\text{Dissim}_{21} = \text{Dissim}_{21} + \alpha_{10}(1 - \beta_1^{F_2}) = 22.5 + 2(1 - (\frac{1}{2})^{0.25}) = 22.82;$$

Rule 10 (Cardinality related Restrictions \geq_{m_k} & \leq_{r_k} & $=_{s_k}$)

This rule handles the expressivity of cardinality related restrictions in estimating our introduced ideal measure. First, we unfold the existing exact cardinality restrictions ($=_{s_k} R_k.T$) to their equivalent minimum and maximum cardinality restrictions ($\geq_{s_k} R_k.T$ & $\leq_{s_k} R_k.T$). Also, if for a property like R_k , there are not restrictions like $\geq_{m_{ik}} R_k.T$ or $\leq_{r_{ik}} R_k.T$ in the description of the $\mathcal{L}'(x_{CN_1i})$ model, we respectively add the restrictions $\geq_0 R_k.T$ or $\leq_{M_2} R_k.T$ to that model to make possible effective comparison between the two models. M_2 is an adjustable factor bigger than all of $r_{jk'}$ which have been used for defining the restrictions like $\leq_{r_{jk'}} R_{k'}.T$ in the description of $\mathcal{L}'(x_{CN_2j})$. In fact, M_2 is used as a replacement for infinite (∞).

$$=_{s_{ik}} R_k.T \in \mathcal{L}'(x_{CN_1i}) \Rightarrow \mathcal{L}'(x_{CN_1i}) := \mathcal{L}'(x_{CN_1i}) \cup \{\geq_{s_{ik}} R_k.T, \leq_{s_{ik}} R_k.T\}$$

$$=_{s_{jk'}} R_{k'}.T \in \mathcal{L}'(x_{CN_2j}) \Rightarrow \mathcal{L}'(x_{CN_2j}) := \mathcal{L}'(x_{CN_2j}) \cup \{\geq_{s_{jk'}} R_{k'}.T, \leq_{s_{jk'}} R_{k'}.T\}$$

$$\text{if } \nexists (\geq_{m_{ik}} R_k.T) \in \mathcal{L}'(x_{CN_1i}) \Rightarrow \mathcal{L}'(x_{CN_1i}) = \mathcal{L}'(x_{CN_1i}) \cup \{\geq_0 R_k.T\}$$

$$M_2 > \text{Max}_{k' \in W} (r_{jk'}), W = \{k' \mid \leq_{r_{jk'}} R_{k'}.T \in \mathcal{L}'(x_{CN_2j})\}, M_2: \text{an adjustable factor}$$

if $\exists (\leq_{r_{ik}} R_k.T) \in \mathcal{L}'(x_{CN_1i}) \Rightarrow \mathcal{L}'(x_{CN_1i}) = \mathcal{L}'(x_{CN_1i}) \cup \{\leq_{M_2} R_k.T\}$

Then, the following equations are used for computing the value which has to be added to the previous value of Dissim_{ij}:

$$\begin{aligned} \text{Dissim}_{ij} &:= \text{Dissim}_{ij} + \alpha_{11} \sum_{(k, k') \in S_1} Z_1(k, k') + \alpha_{12} \sum_{(k, k') \in S_2} Z_2(k, k') + \\ &\alpha_{13} (\sum_{(k, k') \in S_3} Z_3(k, k') + \sum_{(k, k') \in S_4} Z_4(k, k') + \sum_{(k, k') \in S_5} Z_5(k, k') + \\ &\sum_{(k, k') \in S_6} Z_6(k, k') + \sum_{(k, k') \in S_7} Z_7(k, k') + \sum_{(k, k') \in S_8} Z_8(k, k') + \\ &\sum_{(k, k') \in S_9} Z_9(k, k')) \\ &, \quad 0 \leq \alpha_{11} \quad , \quad 0 \leq \alpha_{12} \quad , \quad 0 \leq \alpha_{13} \quad , \quad 1 < \beta_2 \\ , Z_1(k, k') &= \left(\frac{\text{SP}(m_{jk'}, m_{ik})}{\beta_2 \text{RDissim}(R_{k'}, R_k, \infty)} \right) \\ , Z_2(k, k') &= \left(\frac{\text{SP}(r_{ik}, r_{jk'})}{\beta_2 \text{RDissim}(R_k, R_{k'}, \infty)} \right) \\ , Z_3(k, k') &= \text{RSP} \left(m_{ik}, r_{jk'}, \text{RDissim}(R_k, R_{k'}, \infty, 0) \right) \\ , Z_4(k, k') &= \text{RSP} \left(m_{jk'}, r_{ik}, \text{RDissim}(R_{k'}, R_k, \infty, 0) \right) \\ , Z_5(k, k') &= \text{RSP} \left(m_{ik}, s_{jk'}, \text{RDissim}(R_k, R_{k'}, \infty, 0) \right) \\ , Z_6(k, k') &= \text{RSP} \left(s_{jk'}, r_{ik}, \text{RDissim}(R_{k'}, R_k, \infty, 0) \right) \\ , Z_7(k, k') &= \text{RSP} \left(s_{ik}, r_{jk'}, \text{RDissim}(R_k, R_{k'}, \infty, 0) \right) \\ , Z_8(k, k') &= \text{RSP} \left(m_{jk'}, s_{ik}, \text{RDissim}(R_{k'}, R_k, \infty, 0) \right) \\ , Z_9(k, k') &= \text{RSP} \left(s_{ik}, s_{jk'}, \text{RDissim}(R_k, R_{k'}, \infty, 0) \right) + \text{RSP} \left(s_{jk'}, s_{ik}, \text{RDissim}(R_{k'}, R_k, \infty, 0) \right) \\ , \text{SP}(a, b) &= \begin{cases} a - b & \text{if } a \geq b \\ 0 & \text{if } a < b \end{cases} \quad , \quad \text{RSP}(a, b, q) = \begin{cases} \frac{1}{q} & \text{if } a > b \\ 0 & \text{if } a \leq b \end{cases} \\ , S_1 &= \{(k, k') \mid \exists (\geq_{m_{ik}} R_k.T) \in \mathcal{L}'(x_{CN_1i}), \exists (\geq_{m_{jk'}} R_{k'}.T) \in \mathcal{L}'(x_{CN_2j})\} \\ , S_2 &= \{(k, k') \mid \exists (\leq_{r_{ik}} R_k.T) \in \mathcal{L}'(x_{CN_1i}), \exists (\leq_{r_{jk'}} R_{k'}.T) \in \mathcal{L}'(x_{CN_2j})\} \\ , S_3 &= \{(k, k') \mid \exists (\geq_{m_{ik}} R_k.T) \in \mathcal{L}'(x_{CN_1i}), \exists (\leq_{r_{jk'}} R_{k'}.T) \in \mathcal{L}'(x_{CN_2j})\} \\ , S_4 &= \{(k, k') \mid \exists (\leq_{r_{ik}} R_k.T) \in \mathcal{L}'(x_{CN_1i}), \exists (\geq_{m_{jk'}} R_{k'}.T) \in \mathcal{L}'(x_{CN_2j})\} \\ , S_5 &= \{(k, k') \mid \exists (\geq_{m_{ik}} R_k.T) \in \mathcal{L}'(x_{CN_1i}), \exists (=_{s_{jk'}} R_{k'}.T) \in \mathcal{L}'(x_{CN_2j})\} \\ , S_6 &= \{(k, k') \mid \exists (\leq_{r_{ik}} R_k.T) \in \mathcal{L}'(x_{CN_1i}), \exists (=_{s_{jk'}} R_{k'}.T) \in \mathcal{L}'(x_{CN_2j})\} \\ , S_7 &= \{(k, k') \mid \exists (=_{s_{ik}} R_k.T) \in \mathcal{L}'(x_{CN_1i}), \exists (\leq_{r_{jk'}} R_{k'}.T) \in \mathcal{L}'(x_{CN_2j})\} \\ , S_8 &= \{(k, k') \mid \exists (=_{s_{ik}} R_k.T) \in \mathcal{L}'(x_{CN_1i}), \exists (\geq_{m_{jk'}} R_{k'}.T) \in \mathcal{L}'(x_{CN_2j})\} \\ , S_9 &= \{(k, k') \mid \exists (=_{s_{ik}} R_k.T) \in \mathcal{L}'(x_{CN_1i}), \exists (=_{s_{jk'}} R_{k'}.T) \in \mathcal{L}'(x_{CN_2j})\} \end{aligned}$$

Explanation and Justification:

Considering Rule 5, if the probability for $R_k \sqsubset R_{k'}$ is not 0, then $\text{RDissim}(R_{k'}, R_k, \infty)$ is infinite (∞), and if the probability for $R_{k'} \sqsubseteq R_k$ is 1, then $\text{RDissim}(R_{k'}, R_k, \infty, 0)$ is 0. Such a treatment supports and is compatible with our ideal measure, for example, in the case of $R_k \sqsubset R_{k'}$, the min-cardinality restrictions defined over the two properties R_k and $R_{k'}$ are not logically interconnected with respect to the introduced ideal measure and therefore nothing is added to the previous value of Dissim_{ij} as $\text{RDissim}(R_{k'}, R_k, \infty)$ is infinite and as a result the value of $\frac{\text{SP}(m_{jk'}, m_{ik})}{\beta_2 \text{RDissim}(R_{k'}, R_k, \infty)}$ is 0, but in the case of $R_{k'} \sqsubseteq R_k$, the two min-cardinality restrictions are logically interconnected and therefore they have to be compared.

If $\geq_{m_{ik}} R_k.T \in \mathcal{L}'(x_{CN_1i})$ and $\geq_{m_{jk'}} R_{k'}.T \in \mathcal{L}'(x_{CN_2j})$, then if $m_{jk'} \geq m_{ik}$, we have $\text{SP}(m_{jk'}, m_{ik}) = m_{jk'} - m_{ik}$ and if $m_{ik} > m_{jk'}$, we have $\text{SP}(m_{jk'}, m_{ik}) = 0$. In the first case, the value of $\frac{\alpha_{11}(m_{jk'} - m_{ik})}{\beta_2 \text{RDissim}(R_{k'}, R_k, \infty)}$ is added to the previous value of Dissim_{ij} considering the given equation for $Z_1(k, k')$, but in the second case, nothing is added to Dissim_{ij} . Such a treatment supports and is compatible with our ideal measure, because if $\text{Dissim}(R_{k'}, R_k, \infty) \neq \infty$, then in the first case the second model (i.e. $\mathcal{L}'(x_{CN_2j})$) is more restricted than the first one (i.e. $\mathcal{L}'(x_{CN_1i})$) from the perspective of the minimum cardinality restrictions (\geq) defined over the two properties R_k and $R_{k'}$, and this can be interpreted as the less chance for the $\mathcal{L}'(x_{CN_2j})$ model to include more instances which are also included by $\mathcal{L}'(x_{CN_1i})$, and therefore the more the value which has to be added to the previous value of Dissim_{ij} . But, in the case that the first model is more restricted than the second one i.e. $m_{ik} > m_{jk'}$, since our dissimilarity measure has to compute the extent to which the second concept is more restricted than the first concept and not vice versa, nothing is added to the previous value of Dissim_{ij} . The explanation and justification for the function $Z_2(k, k')$ is similar to the explanation given above for the function $Z_1(k, k')$.

The functions $Z_3(k, k'), \dots, Z_9(k, k')$ investigate whether the two models are disjoint or not considering the role cardinality restrictions, and if they are disjoint, Dissim_{ij} is set as infinite. For instance, if $\geq_{m_{ik}} R_k.T \in \mathcal{L}'(x_{CN_1i})$ and $\leq_{r_{jk'}} R_{k'}.T \in \mathcal{L}'(x_{CN_2j})$, then if $m_{ik} > r_{jk'}$, we have $Z_3(k, k') = \text{RSP}(m_{ik}, r_{jk'}, \text{RDissim}(R_k, R_{k'}, \infty, 0)) = \frac{1}{\text{RDissim}(R_k, R_{k'}, \infty, 0)}$, and if the probability for $R_k \sqsubseteq R_{k'}$ is 1, and therefore $\text{RDissim}(R_k, R_{k'}, \infty, 0) = 0$, then $Z_3(k, k')$ and therefore Dissim_{ij} are infinite considering the given equations for them, because in these conditions, the two models do not have any common instance. Hence, such a treatment is also compatible with our introduced ideal measure and supports it. This rule is executed for all pairs of properties like $(R_k, R_{k'})$ over which cardinality related restrictions have been defined. Considering our example, we have:

$$\begin{aligned} \text{Dissim}_{11} &:= \text{Dissim}_{11} + \frac{\alpha_{11}\text{SP}(5, 1)}{\beta_2 \text{RDissim}(S, R, \infty)} + \frac{\alpha_{11}\text{SP}(5, 0)}{\beta_2 \text{RDissim}(S, S, \infty)} = 20.5 + \frac{1 \times (5-1)}{2^1} + \\ &\frac{1 \times (5-0)}{2^0} = 19.82 + \frac{4}{2} + \frac{5}{1} = 26.82; \\ \text{Dissim}_{21} &:= \text{Dissim}_{21} + \frac{\alpha_{11}\text{SP}(5, 1)}{\beta_2 \text{RDissim}(S, R, \infty)} + \frac{\alpha_{11}\text{SP}(5, 0)}{\beta_2 \text{RDissim}(S, S, \infty)} = 22 + \frac{1 \times (5-1)}{2^1} + \\ &\frac{1 \times (5-0)}{2^0} = 22.82 + \frac{4}{2} + \frac{5}{1} = 29.82; \end{aligned}$$

Rule 11 (Enumerations)

$$\left. \begin{array}{l} \exists A = \{o_1, \dots, o_n\} \in \mathcal{L}'(x_{CN_1i}) \\ \exists B = \{o'_1, \dots, o'_{n'}\} \in \mathcal{L}'(x_{CN_2j}) \end{array} \right\} \Rightarrow \text{Dissim}_{ij} := \text{Dissim}_{ij} + \alpha_{14} \left(\frac{|A \cup B|}{|A \cap B|} * \frac{|A - B|}{|A|} \right);$$

, $0 \leq \alpha_{14}$

We also have: $\text{Dissim}_{31} = \text{Dissim}_{11}$, and considering Equation (3.4), $|\text{MHD}(A, CN_1) - \text{MHD}(A, CN_2)| = |3 - 2| = 1 < |\text{MHD}(B, CN_1) - \text{MHD}(B, CN_2)| = |4 - 2| = 2 \Rightarrow \text{MHD}_{ij} = 1$; ($1 \leq i \leq 3, 1 \leq j \leq 1$), So we have: $\text{Dissim}(CN_1, CN_2) = \text{Min}_{1 \leq i \leq 3, 1 \leq j \leq 1} (\text{Dissim}_{ij} + \text{MHD}_{ij}) = 27.82$;

Considering the rules presented in this section, it is clear that each rule is executable, applicable and computable as we have used it for computing the similarity/dissimilarity between the two example concepts. The similarity values resulted from applying the proposed measure can be interpreted as the extent to which the second concept includes instances which are also included by the first concept, because the proposed measure tries to estimate the

introduced ideal measure in computing the similarity/dissimilarity between concepts. In fact, these descriptor specific rules tries to estimate the DD part of the ideal measure (i.e. Equation 2.1) that means Dissim_{ij} (which is computed by the rules) tries to estimate $\text{DD}(\text{CN}_1, \text{CN}_2)$ defined in Equation 2.1.

The probability parameters (i.e. δ_{C_1} , δ'_{C_1} , $\delta_{k'_1 k'_2}$, and $\delta_{k'_2 k'_1}$) used in defining our measure are determined by domain experts if and only if the two compared concepts are from different ontologies. The experts only need to determine these parameters for roles and primitive concepts of the two ontologies and not for complex concepts. Hence, although our method for computing the similarity/dissimilarity between two concepts from different ontologies is semi-automatic but it makes simpler such a complex problem.

The domain experts also need to determine the adjustable factors used in the equations of the rules (i.e. α_i ($1 \leq i \leq 14$), M_1 , M_2 , β_1 , β_2 , and μ). Considering the rules, clearly α_i s are just linear multiplier factors which increase or reduce the effect of various dissimilarity functions used in the rules. There is no any upper limit for the values of α_i ($1 \leq i \leq 14$) since α_i s are linearly used in computing Dissim_{ij} and there is not any upper limit for the values of Dissim_{ij} . Because Dissim_{ij} is ranged from zero (0) to infinite (∞) contrary to $\text{Sim}(\text{CN}_1, \text{CN}_2)$ which is ranged from 0 to 1. By using α_i s, the experts can control the effect of various types of descriptors on the finally computed dissimilarity value. For instance by using α_7 factor, the experts can increase or reduce the effect of property value restrictions on the finally computed dissimilarity value.

β_1 is an adjustable factor used in Rules 4, 8, and 9. Considering these rules and the fact that $0 \leq \beta_1 \leq 1$, it is clear that β_1 has always been used in the form of $\alpha_j(1 - \beta_1^F)$ ($j=4, 9, 10$) when the value of the descriptor specific function (i.e. F) does not have to directly affect the computed dissimilarity value, because we have heuristically recognized the need for such a mechanism in the cases reflected in Rules 4, 8, and 9 in order to correctly estimate the introduced ideal similarity metric. So, β_1 can be used for adjusting this mechanism used in Rules 4, 8, and 9.

β_2 is an adjustable factor used in all the rules handling the property (role) restrictions (i.e., Rules 6, 7, 8, 9, and 10). Considering these rules and the fact

that $1 \leq \beta_2$, it is clear that β_2 has always been used in the form of $\frac{1}{\beta_2^{\text{RDissim}(R_k, R_{k'})}}$ to control the effect of roles dissimilarity value on the value of the corresponding descriptor specific functions. For instance, when $\beta_2 = 1$, the roles dissimilarity value does not affect the value of the corresponding function, but by increasing the value of β_2 , the roles dissimilarity value inversely affects the value of the corresponding function.

M_1 and M_2 are adjustable factors with a relatively big value used in Rules 8 and 10 for the cases in which a specific type of property restriction does not exist in the description of one of the two compared models but it exists in the description of the other model. So, by using the adjustable factors M_1 and M_2 , a balance between the two compared model descriptions is established that leads to an effective comparison.

4. Discussion and Comparison with other Approaches

In our previous research work [25], we extended the previously proposed theories for logic based matching of web services that were based on simple subsumption reasoning. We proposed an ideal semantic similarity metric to extend simple subsumption based similarity metrics in order to include the states in which the two compared concepts overlap but none of them subsumes the other. The introduced ideal metric is more perfect than simple subsumption based ones to be used in the field of web service retrieval since it can increase the recall-based performance of the web service matchmakers which utilize semantic similarity measures for web services matching and composition [25]. But since the introduced ideal measure is not generally actually computable, we heuristically invented a number of computable rules presented in Section 3 in this paper, which collectively try to estimate the ideal measure based on OWL descriptions of concepts in ontologies. Hence, by presenting our proposed measure as a set of computable and applicable rules, we actually demonstrated that it is possible to compute the similarity/dissimilarity between concepts based on the ideal metric introduced in Section 2 [25].

However as discussed in [25], there are two important dimensions along which the conditions can be changed for semantic similarity measurement:

1) ***DL Expressivity Usage*** that is the complexity of concept definitions in ontologies or in other words, how much the expressivity of description logics has

been used for defining or describing concepts in ontologies. As logic based similarity measures compute the similarity between concepts by handling the expressivity of DLs to an extent, ontologies with proper DL Expressivity Usage are needed for fair and complete evaluation of such measures.

2) ***Proportion of Overlapped Concepts*** that is the extent to which there are pairs of concepts in ontologies that overlap but none of them subsumes the other. Generally, when we compare two concepts, there are three possible situations: 1) The two concepts are disjoint i.e., the intersection of them is not satisfiable, 2) One of the two concepts subsumes the other, and 3) The two concepts overlap i.e., the intersection of them is satisfiable but none of them subsumes the other. Considering all pairs of concepts in ontologies, it seems for many of existing ontologies, the proportion of concept pairs belonging to the third category is much less than the proportion of concept pairs belonging to the first and second categories. As some logic based similarity metrics, might measure the similarity/dissimilarity between two concepts based on the extent to which the two concepts overlap, concept pairs from the third category are also needed for fair and complete evaluation of those similarity measures.

Our proposed measure is able to handle the expressivity of DL based ontology languages to a considerable extent. However, the reliability and performance of logic based similarity measures including our proposed measure depends on the “DL Expressivity Usage” of the ontologies in which the compared concepts have been defined and described. So, if concepts are poorly described in ontologies without an effective usage of the expressivity of description logics, using a sophisticated logic based similarity measure like our proposed measure, does not lead to a good performance. On the other hand, since the ability of our proposed measure in handling the expressivity of description logics is limited, if concepts are described in ontologies with a high usage of the expressivity of description logics beyond the scope of the DL Expressivity Usage which can be handled by our measure, then using our proposed measure does not also lead to a good performance. Anyway, using our proposed measure in the scope of the DL Expressivity Usage which can be handled by it (reflected in the rules presented in Section 3) can lead to a good performance. It should be also mentioned that it seems many of the real ontologies are simple and most of the semantic constructs of highly expressive ontology languages such as OWL, have not been used for

building them. Hence, using sophisticated similarity/dissimilarity measures such as one presented in this paper on the ground of such simple ontologies does not intuitively theoretically make any significant difference in comparison with simple measures. It also seems the Proportion of Overlapped Concepts in many of existing ontologies is low. So, using our proposed similarity/dissimilarity measure on the ground of those ontologies does not intuitively theoretically make any significant difference in comparison with simple subsumption based measures since one important facet of our proposed measure is its ability to compute the extent to which two concepts overlap even if none of them subsumes the other.

As demonstrated in our previous research paper [25], the quality (i.e. effectiveness) of DL-based similarity measures which are used in the field web services retrieval can be evaluated by answering the questions about how well they are able to precisely estimate the introduced ideal similarity metric [25]. In Section 3, we have presented the descriptor specific rules of our proposed measure that try to estimate the introduced ideal measure from their particular descriptor-specific perspectives. In fact, the most important difference between our proposed measure and the most of other DL-based measures, which are able to handle the expressivity of DL-based ontology languages to an extent, is that they do not try to directly estimate the ideal similarity metric introduced in Section 2 [25]. Our proposed DL based measure can be compared with other DL based measures proposed in the literature in order to investigate how well it is able to estimate the ideal measure relative to other ones, although some of the proposed measures may not try to estimate the ideal similarity metric at all.

The authors in [28], present a DL-based approach for semantic matching of web services. It seems their proposed DL-based measure, represented as a pseudo code, tries to estimate the introduced ideal similarity metric although they have not exactly specified the semantics of similarity (values) in their research paper. Their proposed measure is not also as flexible and precise as our measure. They represent their similarity/dissimilarity measure in the form of a pseudo code but we have chosen a rule based representation that makes our measure more flexible, extensible, and understandable considering the facts that our measure is essentially much more sophisticated than theirs and we have used a number of adjustable factors which makes our measure more flexible. While their measure considers the restrictions defined on the cardinality of properties, but it handles

them inadequately just by adding constant values (i.e. 1) to the overall distance between two concepts when there is difference between the two compared descriptors in the semantic description of the two concepts. On the other hand, their measure has not been devised in a way to precisely estimate the introduced ideal measure. For instance, if there are restrictions like $\geq_m R.T$ and $\leq_n R.T$ in the semantic description of two compared concepts respectively and we have $m > n$, then the two concepts are disjoint and cannot have any common instance, and therefore their dissimilarity (or distance as used in [28]) must be infinite (∞) according to our introduced ideal measure, but their measure is not able to recognize this state while our proposed measure is able to recognize this state (rule (10) in section 3.3) and some other important similar states. At least the measure proposed in [28] leaves the disjoint states to be recognized by the underlying reasoner although the underlying reasoner may not be able to recognize many of such states. Their proposed measure also handles the value and existential restrictions simply by increasing the distance of the two compared concepts by the value computed for the distance between the two restricting concepts, while our measure is able to handle such descriptors much more precisely in order to estimate the introduced ideal measure (rules (6), (7), and (8) in Section 3.3).

Some of the proposed DL based approaches, such as ones presented in [11] and [12], may handle value and existential restrictions or cardinality related restrictions, but they take a network-based approach to compute the similarity of roles (R and S) within a hierarchy that is defined as ratio between the shortest path from R to S and the maximum path within the graph representation of the role hierarchy where the universal role U ($U \equiv \Delta^I \times \Delta^I$) forms the graph's root. Such a similarity measure for roles cannot specify whether one of the two roles subsumes the other or not, while answering this question is the basis of our measure for the similarity of roles and then the similarity of complex concepts which are partly defined by placing restrictions on the cardinality or values of those roles and as presented in Section 3.3, such a treatment with respect to the roles is necessary for estimating the introduced ideal measure where role restrictions are compared with each other.

Finally, most of the proposed DL-based similarity/dissimilarity measures contrary to our proposed measure are not able to compute the similarity/dissimilarity between concepts from different ontologies.

5. Conclusion and future work

Our research work can be considered as some preliminary efforts in handling the expressivity of DL-based ontology languages for computing the similarity between concepts in the field of web service retrieval. So, we hope that besides the ongoing efforts for building more sophisticated and precise ontologies, such efforts be also continued to lead to more sophisticated semantic similarity measures which can be used in various areas including the field of web service retrieval. However, our proposed measure opens the way for ontology engineers to build sophisticated ontologies with high DL Expressivity Usage and high Proportion of Overlapped Concepts to be used in sophisticated service oriented applications. Because such sophisticated ontologies need to be handled based on the introduced ideal similarity metric so that they can be completely useful in service oriented applications and there is a relatively sophisticated similarity measure like our proposed measure which tries to estimate that ideal metric as precise as possible based on OWL descriptions of concepts in those ontologies.

Our proposed measure is not able to handle the descriptors of roles (properties) such as ones which are described as transitive, symmetric, functional, or inverse functional properties that may affect the logical interpretation of the other types of descriptors. Such descriptors are a part of some highly expressive DL-based ontology languages such as OWL, but handling them based on the introduced ideal measure was beyond the scope of this research and it is left to future works. It should be also mentioned that handling the expressivity of more expressive ontology languages such as OWL 2 is also desired which can be addressed by future works.

It is also needed to find a suitable solution to the problem of matching the semantic descriptions of web services in which our similarity/dissimilarity measure has to be used as a fundamental operation. Finally, another important issue is the efficiency of software system which implements such sophisticated similarity measures and uses them for semantic matching of web services. The efficiency considerations are also left to future works.

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